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Preservice secondary mathematics teachers' views about using multiple representations in mathematics instruction

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Abstract

The purpose of this study is to determine the preservice secondary mathematics teachers' (PSMTs') views about using multiple representations (MRs) in mathematics lessons. 25 PSMTs were participated in this qualitative study by planning and preparing lessons related to mathematics curriculum attainments. Data were obtained from the form of answers to a questionnaire that includes open-ended questions, lesson plans and participant observations about using MRs in mathematics instruction. The findings of the present study indicate that although PSMTs have some concerns about the usage of MRs, they believe that using them was necessary for mathematics instruction.

Keywords: Preservice mathematics teachers, multiple representations, manipulatives.

1. Introduction

Mathematics education research community strongly agrees on the importance of MRs in the development of mathematical understanding (e.g., Goldin, 2002; Schultz & Waters, 2000; Kaput, 1999; Janvier, 1987; Lesh, Post & Behr, 1987). Similarly, NCTM's Principles and Standards for School Mathematics in USA and recently established mathematics curriculum in Turkey emphasize the value of MRs in mathematics instruction (NCTM, 2000; MoNE (The Ministry of National Education, 2007). According to NCTM standards, students should be encouraged to use and create MRs to develop and deepen their understanding of mathematical concepts. They should use them as tools for mathematics learning and also make translations among them (NCTM, 2000). Likewise, mathematics curriculum in Turkey indicates that if students' construction process of mathematical knowledge is enriched with MRs, the instruction environment will provide a meaningful learning for students (MoNE, 2007).

There are several theoretical frameworks including MRs, which are provided by basis of different theories (Çıkla, 2004). One of them, which was suggested by Lesh (1979), describes five distinct types of representation systems that occur in mathematical understanding (as cited in Lesh et al., 1987). These systems were described as (i) real-world situations, (ii) manipulatives, (iii) pictures or diagrams, (iv) spoken symbols, and (v) written symbols. Lesh et al. (1987) put emphasis on translations among these five MRs by preserving the importance of understanding and using them. They state that making translations between and within modes of representations are key factors in mathematical learning and problem solving abilities.

As the related literature indicates, teachers play an important role in students' understanding and learning abilities during mathematics instruction. Wood (2006) points out that the representation preferences of teachers affect

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students when they construct meaning of a mathematical concept. He determines the goal of using MRs, from the teachers' perspective, as to make abstract mathematical ideas comprehensible for students (p.17). As a result, teachers should give opportunities to students to create their own representations, and guide them to see the similarities and differences among different representations (Smith, 2004). At this point, Lesh et al. (1987) suggest teachers presenting ideas in one representational mode and ask students to illustrate, describe or represent the same idea in another mode. Researchers state that teachers can diagnose students' learning difficulties and identify instructional opportunities by this way (Lesh et al., 1987).

There is abundant research considering the effects of using MRs in instructional environments (e.g., Wood, 2006; Çıkla, 2004) or students'/preservice teachers' translation abilities among the representations (e.g., Gagatsis & Shiakalli, 2004). The common property of these researches is that they are dealing with MRs such as graphs, tables, formulas, verbal descriptions or diagrams. Limited numbers of researches, which analyze technology-based representations (e.g., İpek & Baran, 2011; Özmantar, Akkoç, Bingölbali, Demir & Ergene, 2010), manipulatives (e.g., Strom, 2009; Moyer-Packenham, Salkind & Bolyard, 2008; Suh, 2005) or both of them with real-life examples (Bukova-Güzel, 2010) suggest new studies which integrate these representations into mathematics education. In this regard, it was aimed to determine PSMTs' views about using technology-based representations, manipulatives and real world situations in addition to verbal, algebraic and graphical representations in mathematics lessons. Because future teachers are preservice teachers at present, we believe that revealing their views about using MRs in mathematical lessons is an important issue to be considered.

2. Methodology

As stated above, because determining PSMTs' views about using MRs in mathematics instruction was the purpose of the study, we designed the research as a case study.

2.1. Participants and Context of The Study

The secondary mathematics teacher education program takes five years in Turkey. During the program, PSMTs take courses about mathematics, mathematics education, and pedagogy. The 25 PSMTs, who were participated in the study, had already completed most of the courses offered by teacher training program at a state university. The data were collected in Instructional Methods in Mathematics II course, which is related to mathematics education. PSMTs prepared and applied lesson plans related to mathematics curriculum attainments.

A standard lesson plan has some sections such as curriculum attainments, prerequisite knowledge, materials used, classroom organization, guideline for teacher and students activities, and assessment. While PSMTs were designing their lessons they were asked to add some more requirements to these sections. Additional requirements were about integrating technology-based representations (e.g., virtual manipulatives, internet-based videos, pictures, animations, and dynamic mathematics software), physical manipulatives and real-world situations with lesson plans. PSMTs were asked to plan and prepare lessons in trios and apply their lesson plans to the other pre-service teachers during the course. After every application, PSMTs discussed and evaluated the applied lesson plan by the help of the instructor. Figure 1 provides examples of physical manipulatives, which were designed by PSMTs.

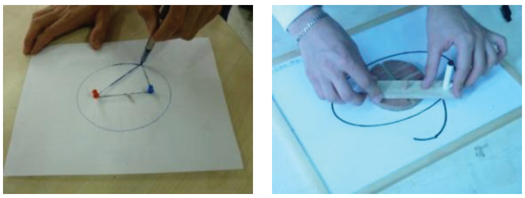

Figure 1: Examples of Physical Manipulatives

2.2. Data Collection Procedure

After finishing all applications, a questionnaire, which includes open-ended questions about using MRs in mathematics instruction, administered to PSMTs. *"What do you think about using technology-based representations, physical manipulatives and real-world situations altogether in your lessons?"* and *"Do you think your experiences about MRs in this course will be reflected on your lessons when you become a mathematics teacher? Please describe your answers with your reason."* can be shown as example questions.

Data were obtained from the form of answers to this questionnaire, lesson plans and participant observations. These observations were focused on PSMTs' experiences while they were using MRs during the lessons.

2.3. Data Analysis

The qualitative data were separated into meaningful units and then coded and categorized according to these units. After finishing the categorizing, the themes were constructed. Besides, during the data analysis the participants were given nick names.

3. Findings and Discussion

PSMTs' views about using MRs in mathematics lesson classified in three themes according to document analysis of the answers to questionnaire, lesson plans, and observation notes. They were identified as (i) necessity of using MRs during mathematical instruction, (ii) some concerns about using MRs in schools, and (iii) some criticisms about the usage of MRs in mathematics lessons. These themes will be clarified in the following sections.

3.1 Necessity of using MRs during mathematical instruction

Most of the PSMTs have stated that using MRs in mathematics lessons was an essential issue for the development of students' mathematical understanding. According to these PSMTs, providing durability of mathematical concepts, arousing interest during the lessons, appealing to the more sense of students, and benefiting from them in the context of subject-matter knowledge were notable advantages of using MRs.

PSMTs indicated that appealing to students' more sense with representations would help students to develop better and permanent concept images. These images would increase the memorability of mathematical ideas. In this regard, most of the PSMTs emphasized the role of physical manipulatives in effective and durable conceptual understanding. Some other PSMTs emphasized the importance of being aware of the relations between MRs was also an important factor in mathematical understanding. They claimed that MRs would facilitate seeing the similarities and differences among representations of the same concept for students. PSMTs' these ideas suited the findings of quite a few researches such as Berthold, Eysink & Renkl (2009), Wood (2006) and Çıkla (2004). Although several PSMTs stated that mathematical ideas would be ideally constructed by students with the aid of bridges between different representations, PSMTs had some difficulties in relating MRs to each other during their lessons. The findings of Mallet (2007) also support this observation by indicating that the links among the MRs are often not established during instructions by the teachers. At this point it can be said that, as Ainsworth (1998) states, providing the translations within and among the MRs is not an easy matter for students, PSMTs need to be more careful about this difficulty during their lessons.

The other point PSMTs highlighted was the motivational property of MRs. They noted this property by saying that MRs increased the level of motivation and readiness of students. One part of PSMTs pointed especially the incentive role of technology-based representations out during the instruction. Some of them claimed that one of the obstacles in engaging students' attention was the abstract form of mathematical structures and especially real-world situations would play a crucial role in showing the engaging face of mathematics. As a result, when they became a teacher they would use real world situations in their lessons. These PSMTs were concentrating on activities about real-world situations more than other MRs during the course. So, it can be said that teachers' preferences are based on their sense-making features of representations.

A surprising finding related to the usage of MRs was about PSMTs' development of subject-matter knowledge. PSMTs indicated that especially the preparation process of physical manipulatives led them to realize their misconstructions about the curriculum concepts. One of them noted this factor by stating that seeing these different representations altogether helped her to think about the properties of mathematical concepts and ideas that she had never been realized before. Likewise, other PSMT supported this idea by these sentences:

"*I think designing lessons by using MRs keeps teacher alive about her/his professional development. S/he can't stop thinking about the instruction process outside the classroom. May be it is not very easy for teachers to prepare them but I think this is just what we need in schools.*"

To sum up, providing durability of mathematical understanding, arousing interest and making contributions to professional development were notable advantages of using MRs for PSMTs. stimulate

3.2 Some concerns about using MRs in schools

Although most of the PSMTs believed the advantages of using MRs in lessons, an important number of them had some concerns about this idea. These concerns were about students' and parents' expectations about university entrance exam, time problems, the technological infrastructure of schools, and the number of students in a classroom.

PSMTs stated that, according to their observations in schools, students and parents had expectations from teachers such as preparing the students only to the university entrance exam. So, they would avoid from using MRs in their lessons. Similarly, some PSMTs mentioned that the most challenging factor about using MRs was using the lesson hours efficiently. It was observed during the course that PSMTs had difficulty about completing their lessons in time. They needed to be more organized about using MRs depending on the lesson hours of attainments.

PSMTs' another concern about using MRs was related with resources of schools and the number of students in classrooms. In addition, they mentioned that the technological infrastructure in classrooms plays a key role for using MRs properly. One of the PSMT expressed these findings as follows;

"*If there is not enough technological equipment in a classroom, trying to use MRs becomes a big problem for teachers. Unfortunately another important issue is the class size. I don't believe that I can use them effectively in a crowded class. I'm worried about using them because of the setting conditions.*"

It can be stated that PSMTs' concerns about using MRs are based on generally present conditions of schools and expectations of students about university entrance exam.

3.3. Criticisms about the usage of MRs in mathematics lessons

As stated above, albeit most of the PSMTs believed the importance of using MRs, a small number of them asserted some criticizes about using them. They thought that teachers couldn't use every kind of representations for every mathematical subject because MRs could be boring and distracting sometimes. In other words, using them would become an ordinary task after a while and could cause discipline problems among the students. It can be said that these PSMTs dealt with MRs as a motivator and when students got used to them, lessons would not be as interesting as before. However, some PSMTs stated that if students got used to them, it would facilitate overcoming time problems in lessons. As a result, some PSMTs believed that practicing MRs in lessons was an advantage but some of them considered this practice was a disadvantage for instruction.

The other criticism about integrating MRs with mathematics lessons was about preparation process. According to these PSMTs, the preparation process of lessons was tiring and difficult for teachers. Even though they agreed that MRs had a vast number of advantages, they said designing lesson plans by adhering to MRs would be a difficult task, especially for present teachers in the system. PSMTs noted that the preparation process required especially several technological competencies and teachers in schools were not ready to take this responsibility as they observed. Fortunately it can be said that, as Juersivich, Garofalo & Fraser (2009) states, present preservice teachers are more familiar to technologic culture than present teachers at schools therefore they can be easily adapted to use technology in their lessons.

4. Conclusions

The findings of the study indicated that PSMTs have different views about using technology-based representations, physical manipulatives, and real-world situations in addition to verbal, algebraic and graphical representations in mathematics lessons. Despite the fact that most of PSMTs thought using MRs was necessary for mathematical instruction, some of them had some concerns about using them in schools related to schools' conditions and student expectations about the university entrance exam. Moreover, a small number of PSMTs criticized the usage of MRs in mathematics lessons with regard to present teachers' proficiencies and students' position of being accustomed to them.

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